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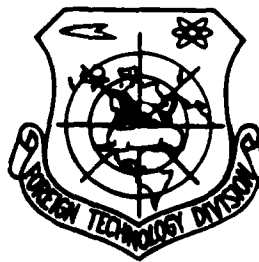
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STRUCTURAL MECHANICS

Contemporary State and Prospects for Development

(Selected Portions)

by

V.V. Bolotin, I.I. Gol'denblat, A.F. Smirnov

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

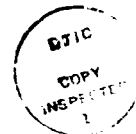| Russian	English
rot	curl
lg	log

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STRUCTURAL MECHANICS.   Contemporary state and prospects for
development.

V. V.   Bolotin, I. I.   Gol'denblat, A. F. Smirnov.

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PREFACE.

In resolution of XXIV Congress of Communist Party of Soviet Union it is indicated that "decisive condition of increasing efficiency of public production is acceleration of scientific-technical progress. Should be expanded fundamental scientific investigations, more completely utilized achievements of science and technology..."

In this respect high value has compilation of surveys/coverage of the contemporary state of different branches of science and engineering, surveys/coverage, which give the possibility to reveal the prospects for the development of these branches and increase in their effectiveness.

In recent years are published many works on structural mechanics and its application to calculation of structures, aircraft, ships and machines. However, among these investigations are still insufficient the works of survey character, which give representation about the level of the interesting us field of science as a whole.

In this book contemporary state of some problems of structural mechanics is examined; in this case are isolated those tasks, which, in the opinion of authors, did not draw proper attention of researchers.

Survey/coverage in no way pretends to completeness; authors paid main attention to works, close to their scientific interests. Are in detail examined such sections of structural mechanics as the theory of the elastoplastic calculation of constructions, statistical dynamics and the theory of the reliability of constructions, mechanics of constructions from the composite materials. Special chapter is dedicated to the application of electronic computers in structural mechanics. Russian Translation. (JPS)

In bibliographies, given at the end of corresponding chapters, are included only some transactions, characteristic for directions in question. The detailed enumeration of Soviet works in the region of structural mechanics is contained in the collections edited by I. M. Rabinovich "Structural mechanics in the USSR in forty years (1917-1957)" (Gosstroyizdat, 1957) and "Structural mechanics in the USSR in fifty years (1917-1967)" (Stroyizdat, 1968).

First chapter of book is written by I. I. Gol'denblat in co-authorship with V. L. Bazhanov and by V. A. Kopnov, second and third chapters by V. V. Bolotin, the fourth - by A. F. Smirnov.

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Chapter II.

MECHANICS OF COMPOSITE MATERIALS AND CONSTRUCTIONS FROM THEM.

1. Introduction.

Artificial materials, which consist of two or several components with different physicomechanical properties, extensively are used in contemporary technology. These materials are called composition, composite or simply composites. Components can be elastic, viscoelastic, highly elastic and even gaseous (as example of composites with the gaseous components serve foam plastics and other porous materials). Via the proper selection of components, their relative contents and structure of composite it is possible to create the new materials, in which is combined high strength and rigidity with other valuable qualities: relatively light specific weight, low heat conductivity, durability with respect to the aggressive media, etc. Composites can be both macroscopically isotropic and macroscopically anisotropic. The character of the artificial anisotropy of composites can change in very wide limits. The application of anisotropic composites will be especially effective, if it will be possible to coordinate the stress fields and strains with the fields of mechanical characteristics.

Strictly speaking, almost all materials, used in building and machine building, should be related to the category of composites. To

them belong the metals and the alloys, which possess a heterogeneous
polycrystalline structure, concrete, brick and other silicate
materials and, it goes without saying, wood. Last material serves as
example of the composite of natural origin. As V. A. Kargin [39]
indicated, the principle of contemporary composite materials was
borrowed from nature. The majorities of animal and plant tissues
(skin, bone, the muscular tissues of animals and almost all plant
tissues) possess the composite structure, which was manufactured in
the process of prolonged selection. Subsequently, however, we will be
restricted to the examination of artificial reinforced materials.

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By reinforced materials we will understand composites, one of
components of which possesses high mechanical strength and rigidity in
comparison with another. From the second component usually are
required light specific weight, high adhesive and cohesive properties.
The first component, called the reinforcing material or filler, is
intended for the transmission of basic flux of forces in the
construction. The designation/purpose of the second component
(bonding agent) lies in the fact that to ensure connection between the
single particles of composite. It is essential that the
size/dimension of the structure of composite (for example, the
diameter of filament, the size of particle, etc.) would be small both
in comparison with the significant dimension of construction or its
element and in comparison with the distances, at which the smoothed
macroscopic stress field and strains is changed by the noticeable

value. From this point of view reinforced-concrete constructions must
be considered as consisting of substantially different materials.
Exception is perhaps the case of dispersed reinforcement by
sufficiently thin wire. This reinforced concrete already can be
treated as composite material.

As example of reinforced composite materials serve filled rubber
and rubber-cord materials, wood laminate plastics, Getinax
[laminated paper-Bakelite insulating material] and Textolite, which
have already long ago been used in technology. The number of most
important reinforced composites includes glass-fiber-reinforced
plastics, in which the high strength of glass fibers and the high
adhesive properties of phenol, polyester/polyether, epoxy and other
polymeric resins are utilized. Although first attempts of the
implementation of glass-fiber-reinforced plastics were made even in
the pre-war period, their wide acceptance occurred only in last decade
because of the successes of technology of polymers and synthetic
resins. Besides the high mechanical strength, dielectric strength,
relative lightness and other valuable physicomechanical qualities, the
glass-fiber-reinforced plastics possess a good manufacturability.
With comparatively simple equipment, the small labor inputs and the
minimum wastes it is possible to manufacture the constructions of very
intricate shape and even with the specified distribution of
physicomechanical properties.

Glass-fiber-reinforced plastics are used in most varied areas of

technology. High strength, stability to the aggressive media and relatively high heat resistance ensured the implementation of glass-fiber-reinforced plastics in the chemical industry. From the glass-fiber-reinforced plastics via coil/winding are manufactured the reservoirs, intended for the work under the pressure, tubes/pipes, elements of fittings, etc. In machine building the glass-fiber-reinforced plastics are utilized for manufacturing the diverse parts. Very widely glass-fiber-reinforced plastics are applied in aviation and rocket engineering; from them are manufactured the missile bodies, rocket thrust chamber, control surface, fairings, etc. The hulls of surface and underwater ships, body of vehicles, etc are manufactured from the glass-fiber-reinforced plastics.

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Application of glass-fiber-reinforced plastics in building was begun about 20 years ago. First they were used as the enclosing and decorative elements, and recently - as the frameworks. Obstacle for the wide propagation of glass-fiber-reinforced plastics in the building thus far still serves their relatively high cost, and also insufficient service life, which is expressed in aging of glass-fiber-reinforced plastics. At the same time the high and specific manufacturability of glass-fiber-reinforced plastics, the light weight of structural elements open great possibilities for their use in the architecture, especially during the creation of new, nontraditional architectural forms. By this is explained the fact that striking examples of the application of glass-fiber-reinforced

plastics and building relate to the constructions of exhibition character. According to the data of L. Gapl [32], the glass-fiber-reinforced plastics were used for the frameworks in 1955 in one of the exhibition pavilions of CSR. In cited book [32] it is possible to find the series of other examples of the application of glass-fiber-reinforced plastics for the shells, the cupolas, the roofing coatings, the elements of volume of building and whole habitable houses.

At present successfully are developed/processed other composite materials [11, 26]. On the basis of polymeric bonding agents are created the materials, in which as the reinforcing elements are utilized the filaments of boron and carbon, carbide of silicon and other ceramic substances, single-crystal metallic filaments, etc. On the other hand, are conducted the research on the use as the binders of light metals. Are marked out achievements in the region of technology of self-reinforced metals. Are noted achievements in the region of technology of self-reinforced metals, in which the role of the reinforcing elements perform single-crystal filaments from the same metal.

Wide acceptance of composite materials requires creation of mechanics of their deformation and destruction. The need of developing this theory is additionally strengthened by the fact that the properties of material itself can to the known degree be designated in the process of planning. Thus, theory is necessary not

only for calculating the constructions from the prescribed/assigned material, but also for planning the material itself.

In this chapter survey/coverage of some directions in development of structural mechanics of reinforced composite materials and constructions from these materials is given. This survey/coverage does not pretend to the completeness both with respect to the illumination of all aspects of problem and in bibliographical sense. The numerous information about the glass-fiber-reinforced plastics and other composite materials can be found in works [4, 11, 26]. Soviet work in the mechanics of the reinforced plastics is illuminated in the books of P. M. Ogibalov and Yu. V. Suvorova [63], V. L. Bazhanov, I. I. Gol'denblat, V. A. Kopnov, A. D. Pospelov and A. M. Sinyukov [5], Yu. M. Tarnopol'skiy and A. M. Skudra [83], Yu. M. Tarnopol'skiy and A. V. Rose [86], A. L. Rabinovich [72], G. A. van Fo Fy [89], V. I. Korolev [41], V. B. Meshcheryakov, A. K. Sborovskiy and A. Ya. Goldman [51], [34].

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2. Macroscopic characteristics of composite materials.

It is possible to mark two ways of construction of mechanics of composites. First path - purely phenomenological. It assumes the direct use of already known equations of the theory of elasticity, theory of creep and so forth for the anisotropic body. In this case mechanical constants are determined on the basis of the laboratory

tests of samples made from composite material.

Alternate path of construction of theory of deformation and destruction of composites is based on structural considerations. Target lies in the fact that to connect the mechanical characteristics of composite with the mechanical characteristics of components, the coefficients of reinforcement, the sizes/dimensions of the reinforcing elements and other structural parameters. Even when this way reduces to the already known equations of the theory of elasticity and theory of creep for the anisotropic media, it proves to be more preferably both with the theoretical and from a practical point of view. It makes it possible to naturally link questions of elasticity, creep and strength, to predict the mechanical properties of composites concerning the mechanical characteristics of components, to solve the problems of the optimum planning of the reinforced materials, etc.

In examination of composites it is possible to introduce two and even three different levels of description, that are characterized by length scale. Lower level - this is the level of structural heterogeneity. Its scale, designated subsequently through h, is equal to the significant dimension of the reinforcing elements - diameter of grain, to the diameter of filament or to the thickness of the reinforcing layer. As the following level of examination serves that, on which is possible the replacement of a heterogeneous material by locally uniform equivalent in a sense material. The appropriate scale let us designate through H. finally, summit level - this is the

level, whose scale Λ is equal to the significant dimension of article
and (or) to the distance, on which the smoothed stresses and strains
change to the noticeable value. And in the absence of too sharp/acute
concentrators it is possible to consider that for the sufficiently
large/coarse articles $h \ll H \ll \Lambda$ (Fig. 5).

Solution of problem decays into several stages. During the first
stage (level h) the problem for the substantially heterogeneous
material is solved. Then via the averaging of stresses and strains in
the volume of order H^3 are calculated the macroscopic characteristics
of composite material. Following stage - solution of boundary-value
problem for the body with the obtained macroscopic characteristics
(level Λ). If target consists of the determination of the smoothed
stress fields, strains and displacements, then the solution of problem
concludes on this. But if it is necessary to find structural stresses
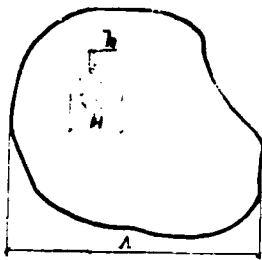and strains, then it is necessary to return to level h.

Fig. 5.

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In present section let us pause at computation of macroscopic characteristics of composite materials, which corresponds to examination of composite at level h with transition/transfer to level H. The methods discussed here are applicable for determining the elastic and viscoelastic characteristics of composites, coefficients of thermal conductivity, coefficients of the thermal expansion and other characteristics, which link the smoothed fields of the physicomechanical parameters. The forecast of the strength characteristics of composites presents some additional difficulties, about which will be said somewhat below.

Theory of composite materials rises from from classical work of Voight [105]. True, Voight dealt in essence with polycrystals, posing the problem of the determination of the properties of polycrystal according to the known properties of crystallites. But, on one hand, polycrystal can be considered as special type composite, in which the heterogeneity is created because of the difference in the orientation

of crystallites. On the other hand, the principles, for the first
time formulated by Voight, became the basis of further development of
the theory of composites. Voight proposed to calculate the parameters
of the properties of composites via the averaging of the corresponding
parameters of composites. Voight conducted the averaging of the
parameters, which are described by the tensors of the second, third
and fourth ranks. In this case it revealed/detected that the
parameters of composite are, generally speaking, different depending
on whether are averaged the components of straight/direct or
reverse/inverse tensor in the relationships/ratios between the
generalized forces and the generalized displacements. Later Reuss
[103] in connection with polycrystals proposed to average the
components of reverse/inverse tensor. At present the method of
computing the parameters of composite via the averaging of
straight/direct tensor is called Voight's method; the opposite method
is called the Reuss method. For example, for the macroscopically
isotropic composite, which is the mixture of isotropic components with
the moduli of shift/shear G', G" ,..., averaging according to Voight
gives:

$$G_V = \psi' G' + \psi'' G'' + ... \qquad (1)$$

Here ψ', ψ'', ..., — relative volumetric contents of components.
Using averaging on Reuss, we find that

$$G_R = \left(\frac{1}{\psi' G'} + \frac{1}{\psi'' G''} + ... \right)^{-1}. \qquad (2)$$

Disagreement between two methods of averaging will be more

considerable, the more moduli/modules of components are distinguished. Dependences (1) and (2) are shown in Fig. 6 in connection with two-component mixture. In this case it is accepted that $G'=10G''$.
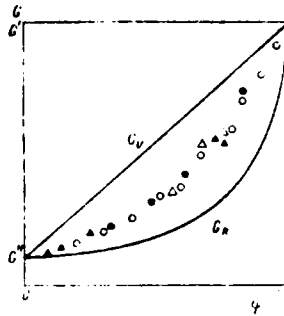
Fig. 6.

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Voight's method corresponds to assumption about uniformity of field of generalized displacements, while method of Reuss - to assumption about uniformity of field of generalized forces. On the basis of the energy considerations, Hill [98] showed that Voight's method gives upper limit for the elastic constants of composite, and the method of Reuss - lower limit.

In recent years was undertaken series/row of attempts obtain narrower evaluations for elastic constant composites. Different variation approaches were developed for this. The discussion of these approaches can be found in the articles of Hill [98, 99], Khashin [97], etc. One should note, however, that the refined evaluations, which give the possibility to consider mutual arrangement and the form of the reinforcing elements, nevertheless prove to be insufficiently precise. Therefore in the mechanics of composites the method of averaging as before most frequently is used. But in contrast to the

initial work of Voight and Reuss averaging is conducted in combination
with some hypotheses about the stress fields, displacements and
strains; the real character of interaction between the elements is
considered with the formulation of these hypotheses. This improved
approach makes it possible to obtain the results, which are
satisfactorily coordinated with experimental data and therefore
sufficient for the technical applications.

Most reliable results are obtained for laminar composites.
Because of the clear character of interaction between the components
of laminate here it is possible to formulate completely reliable
hypotheses about the stress fields, displacements and deformations.
These hypotheses are analogous to the known hypothesis of
Kirchhoff-Love from the theory of plates and shells. The first work
in the mechanics of laminar composites should be considered the
article of S. G. Lechnitzsky [44], in which the task about the
deformation of the elastic plate, which consists of anisotropic layers
with different properties, was examined. The works of V. L. Biderman
[7], A. L. Rabinovich [72], Yu. M. Tarnopol'skiy with co-authers [83,
86] and others are dedicated to a question of the determination of the
mechanical properties of laminar composites. The theory of
anisotropic and laminated panels and shells was developed/processed by
S. G. Lechnitzsky [46], S. A. Ambartsumyan [2, 3], E. I. Grigolyuk
and P. P. Chulkov [35], V. I. Korolev [41], V. N. Moskalenko and Yu.
N. Novichkov [52], etc. The theoretical results, which relate to the
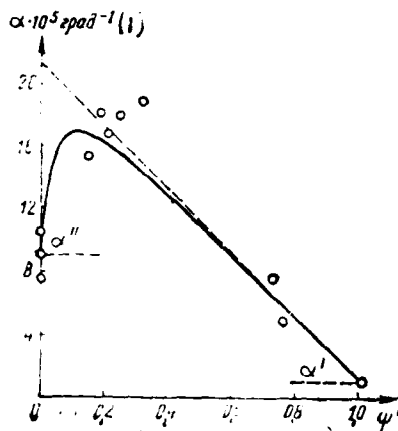laminar composites, repeatedly were compared with experimental data.

Fig. 7.

Key: (1). deg⁻¹.

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As example [24] Fig. 7 gives the dependence of the coefficient of
thermal expansion α in the direction, orthogonal to reinforcement, on
the relative content of the reinforcing material ψ'. Theory and
experiment show that in a certain interval of the variation ψ' the
coefficient of the thermal expansion of composite can exceed the
coefficient of thermal expansion of each of the components.

Interaction of elements of fibrous composites carries more
complicated character. But also here extensively is used principle of
averaging. Together with the approximate approaches, are utilized the
more advanced methods, based on the solution of the corresponding
problems of the theory of elasticity, viscoelasticity and the like for
the inhomogeneous medium. Thus, assuming all filaments identical,

which have the form of circular cylinder and arranged/located in the
correct order, it is possible to reduce the problem about the
deformation of composite to the well studied task of the theory of
elasticity about the deformation of elastic body with the regular
elastic inclusions. The problem indicated is solved after, it is
possible to calculate the effective elastic constants of composite.
This method was used by G. A. van Fo Fy and G. N. Savin [28, 29, 89].
The disagreement between the results, which gives the method of
averaging and a precise method, is usually sufficiently large,
especially if the elastic constants of components strongly differ.
But nevertheless these disagreements, as a rule, do not exceed the
natural scatter of experimental data, but transparency of the method
of averaging and simplicity of its formulas give great practical
advantages to it.

Comparison of results, to which they lead assumptions of
different authors, and also some experimental data, it is possible to
find in review paper of Chamis and Sendetskiy [91].

Stochastic composites occupy special position among composite
materials. Virtually any composite - stochastic in the sense that the
property of components, their arrangement/position and concentration -
random parameters. For example, in unidirectional fibrous material
are observed the scatter of the properties of filaments, their low
bending, caused by the imperfections of technology, local
disturbances/breakdowns of adhesion, shrinkage cracks, etc. Other

materials are specially designed as stochastic. As an example can serve stochastic fiberglass materials, filaments or strands in which are specially entangled in order to obtain quasi-isotropic and quasi-homogeneous material. Stochastic materials are noted by the increased scatter of characteristics, by the increased sensitivity to the scale factor.

In principle method of averaging is applicable to stochastic composites. However, it here proves to be that the transition/transfer from stochastic medium to the equivalent homogeneous medium requires the examination of the correlation properties of composites. In connection with polycrystals to this it was for the first time indicated by I. M. Lifschitz and by L. N. Rozentsveyg [47]. In this article the task about the determination of the elastic constants of polycrystal is examined by the method, analogous to Born approximation in quantum mechanics. Stochastic heterogeneity is considered low, and the solution searches for in the form of series/rows according to the degrees of the low parameter.

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In this way I. M. Lifschitz and L. N. Rozentsveyg found the first correlation correction to elastic constants, calculated according to Voight's method. At present investigations in the mechanics of stochastic composites and polycrystals are conducted by broad front. Let us point out the works of S. D. Volkov and V. Ya. Dolgikh [31], V. A. Lomakin [49], A. G. Fokin and T. D. Shermergor [88], L. P.

Khoroshun [90], V. V. Bolotin and V. N. Moskalenko [23]. In the work of the latter/last authors is proposed the model of the composite material, which possesses the property of strong isotropy in the sense that its macroscopic characteristics do not depend on the multipoint correlation functions of local properties. For this model it was possible to conduct the summation of infinite series and to obtain final formulas for the macroscopic coefficients of thermal conductivity, diffusion, elastic, viscoelastic and thermoelastic characteristics. Is carried out the comparison of the results, which are obtained for the strongly isotropic composite, with the results, which give the methods of Voight, Reuss, of self-congruent field, and also Born approximation. Further development of the theory stochastic composites requires the study of the highest correlation functions for the local characteristics. In this case both the theoretical study of the corresponding probability fields and the statistical analysis of the structure of real composite materials is desirable.

Purpose of above-indicated works consisted of determination of macroscopic characteristics of composites in connection with classical models of continuous medium, which are utilized in theory of elasticity and in structural mechanics. However, it is not previously known, will be the macroscopic behavior of composite material satisfactorily described within the framework of classical representation. Thus, natural to expect that in the composite materials will be observed effects of the type of those, which are discussed in the generalized theories of continuous medium (for

example, in the moment theory of elasticity). Therefore is of
interest discussion of such methods of the construction of the
mechanics of the composites, which are not connected with the in
advance selective models of classical theory. In articles [13-15] the
method of the construction of the theory of composite media, based on
the model considerations, is proposed. This method makes it possible
to obtain the equations of the mechanics of composites at level h or
H, to formulate boundary-value problems and variation principles in
these levels and to incidentally establish connection/communication
between the macroscopic characteristics of composite and the
characteristics of components. This method can be realized either in
discrete/digital [13] or in the continuous [14] version. The latter
is based on the principle of energy smoothing (unsuccessfully named
initially the" principle of smearing"). Let us pause at the method
indicated in somewhat more detail.

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Let us examine inhomogeneous medium, which consists of
connecting/cementing material and large number of reinforcing elements
of increased strength and rigidity. Both connecting/cementing and
reinforcing materials can have the arbitrary flow properties: to be
elastic, viscoelastic, viscoelastic plastic, etc. The reinforcing
elements can be both the one-dimensional (filament or rods) and
two-dimensional (the membrane/diaphragm, plate, shell, two-dimensional
latticed systems).

As basis let us place following two assumptions: (1) rigidity of material of reinforcing elements substantially exceeds rigidity of connecting/cementing material; (2) thickness of reinforcing elements and distance between adjacent elements are low both in comparison with significant dimensions of body and in comparison with distances, at which functions, which characterize stress-strain state of reinforcing elements, are changed by noticeable value.

Formulated assumptions make it possible to express all parameters of stress-strain physical form through parameters, which characterize stress-strain state of reinforcing elements. Thus, if the reinforcing elements are the plates, whose deformations are described by Kirchhoff-Love hypothesis, then the state of each plate can be described by three functions - the displacements of the points of median surface $u_\alpha (x, y), v_\alpha (x, y) w_\alpha (x, y)$. Knowing these displacements for all reinforcing plates ($\alpha=1, 2...., $ n), it is possible to find the distribution of displacements in each layer of the connecting/cementing material, limited by two adjacent plates. If the reinforcing elements are the system of the rods, whose deformations are described by Bernoulli's hypothesis, then for the assignment of displacements at any point of composite it suffices to know on four functions for each reinforcing element - three displacements of the points of its axis/axle $u_\alpha (x), v_\alpha (x), w_\alpha (x)$ and angles of rotation of section/cut around axis/axle $\varphi_\alpha (x)$. Distribution of displacements in the connecting/cementing material is found in this case from the solution of a certain contact problem with the displacements, assigned

on the interfaces of the connecting/cementing and reinforcing
material.

Consistently accomplishing/realizing method of describing state
of composite given above, we will arrive at system of very large (3n
or 4n, where n - number of elements) number of equations relative to
functions, which describe state of each of reinforcing elements.
These equations prove to be differential on to the variables x and y
(or, in the case of linear elements, only on x) and difference
equations - on the indices, which are assigned to the reinforcing
elements. The following step consists of the use of the fact that the
number of reinforcing elements is very great and that the
corresponding functions are changed sufficiently slowly upon transfer
from one reinforcing element to another. This makes it possible to
approximate finite, although very large set of functions one or two
coordinates by several functions of three independent coordinates u(x,
y, z), v(x, y, z), w(x, y, z) and φ (x, y, z). This replacement we
will call "smoothing". Instead of the system of the large number of
differential-difference equations we come to the system of three or
four differential equations, which describe the deformation of certain
quasi-homogeneous medium [13].

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If for conclusion/derivation of basic equations is utilized
variation principle, then transition/transfer to equivalent
quasi-homogeneous medium can be conducted in earlier stage. Namely

the functions of three coordinates, which approximate the distribution

of displacements in the reinforcing elements, can be introduced

already during the compilation of the minimized functional. This

"energy smoothing" leads to the same final results, as the replacement

of differential-difference equations by differential equations with

the large number of independent variables [14].

If components are elastic and if rigidity of reinforcing elements

during bending and torsion is considered, then principle of smoothing

leads to certain version of moment theory of elasticity for

anisotropic medium. · The graphic scale, which plays the significant

role in the moment theory of elasticity, in this case will be of the

order

$$\lambda \sim h \left(\frac{E'}{E''} \right)^{1/2} , \tag{3}$$

where h - scale of the reinforcing element; E' - the characteristic

modulus/module of the reinforcing material; E'' - the characteristic

modulus/module of the connecting/cementing material. If $E' >> E''$, then

$\lambda >> h$. Thus, in the reinforced materials we meet with such situation,

when the scale of moment effect can prove to be sufficiently to large

in comparison with the scale of structure. This means that there is a

region of the application of moment theory, free from the internal

contradictions of the type of those, with which we meet in connection

with the mechanics of polycrystalline materials. Moment effects in

the composite materials were examined in works [15, 49, 57].

Principle of smoothing makes it possible to obtain equations,

which are not encountered in classical continuum mechanics. Let us examine the simplest case of laminar medium with the flat/plane isotropic reinforcing elements (Fig. 8). In addition to hypotheses for the laminar medium, which were formulated above, let us introduce assumption about the fact that the elongations in the direction of normal to the reinforcing layers can be disregarded/neglected. The principle of energy smoothing reduces to equations [14]:

$$\left.\begin{array}{l} \dfrac{\partial^2 u}{\partial x^2} + \nu\,\dfrac{\partial^2 v}{\partial x \partial y} + \dfrac{1-\nu}{2}\left(\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^3 v}{\partial x \partial y}\right) + \chi\,\dfrac{\partial^2 u}{\partial z^2} = 0; \\[2mm] \dfrac{\partial^2 v}{\partial y^2} + \nu\,\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{1-\nu}{2}\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y}\right) + \chi\,\dfrac{\partial^2 v}{\partial z^2} = 0; \\[2mm] D_{+}\,\Delta\Delta w - \Delta w \displaystyle\int_{-H_1}^{H_2} \dfrac{G^{\cdot} c}{s}\,dz - \displaystyle\int_{-H_1}^{H_2} \dfrac{G^{\cdot} c}{s}\left(\dfrac{\partial^2 u}{\partial x \partial z} + \dfrac{\partial^2 v}{\partial y \partial z}\right)dz = q. \end{array}\right\} \qquad (4)$$

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Here D_+ - total flexural rigidity of bundle; χ - dimensionless parameter, which characterizes relationship/ratio between deformation of reinforcing and bonding layers:

$$D_{+} = \int_{-H_1}^{H_2} \frac{E' h^3\,dz}{12\,(1-\nu^2)c}; \qquad \chi = \frac{G^{\cdot} c^2}{s} \cdot \frac{1-\nu^2}{E' h}.$$

Furthermore, ν - Poisson ratio of reinforcing material; h — thickness of reinforcing layers; s - thickness of bonding layers; $c = h + s$; $H = H_1 + H_2$ - thickness of bundle; q - intensity of total load, applied along the normal to layers.

Equations (4) are system of integrodifferential equations

relative to three functions u(x, y, z), v(x, y, z) and w(x, y, z). It
is possible to obtain them from the equations of the moment theory of
elasticity for a certain anisotropic medium by integrating one of the
equations from - H_1 to H_2. In work [14] system (4) is obtained from
the variation principle with the use of kinematic and static
hypotheses. Natural boundary conditions on surface of x=const,
strainless, take the form:

$$
\left.
\begin{aligned}
\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= 0
\end{aligned}
\right\} (-H_1 \leqslant z \leqslant H_2);
\tag{5}
$$

$$
\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0;
$$

$$
D_+ \left[\frac{\partial^3 w}{\partial x^3} + (2 - \nu) \frac{\partial^3 w}{\partial x \partial y^2} \right] - \int_{-H_1}^{H_2} \frac{G^\circ c}{s} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) dz = 0.
$$

To generalization and to application of theory of composites in
discrete/digital version and theory, based on principle of smoothing,
are dedicated works [16-21, 30, 36, 52, 53, 55-61, 64-70, 74-76,
78-86], etc. Some these works examine in one of the subsequent
sections. Here we will pause at articles [17, 18], where the theory
applies to stochastic composite media. In article [17] the problem
about the evaluation of effect on the properties of the unidirectional
fibrous and laminates of the low random bending of the reinforcing
layers is posed. Fundamental equations for medium are obtained on the
basis of the principle of smoothing. Equations were solved by
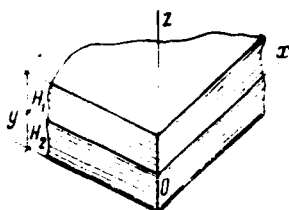spectral method on the assumption that the initial bending are
sufficiently low.

Fig. 8.

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It was shown that the values of elastic constants for the reinforced medium with the initial inaccuracies depend substantially on the base of measurement, being decreased with increase of base. In this case on the base of order $\Lambda>>H$ (H - the scale of inaccuracies) the moduli of elasticity are reduced by 20-30% even with the very low inaccuracies (Fig. 9). Different behavior of material during the elongation and the compression in the direction of reinforcement was discovered also. Difference becomes especially noticeable with compressive stresses, close to the breaking stresses of the local bulge (this breaking stress it is of the order the modulus of the shift/shear of bonding agent). Fig. 10 gives the form of the diagram of deformation, constructed taking into account the enumerated effects.

Theoretical conclusions of article [17] obtained experimental confirmation in works of Yu. M. Tarnopol'skiy, G. G. Portnov and I. G. Zhigun [84-86].

Generalization of theory to viscoelastic materials was given in articles [17, 18]. In this case viscoelastic analogy was utilized. Let us give the approximation formula for averaged on the large base deformation $e_{\alpha\beta}^{\cdot}$ (t), which corresponds to the stationary quasi-homogeneous planar-stressed state with nominal stresses/voltages of $\sigma_{\alpha\beta}$:

$$e_{\alpha\beta}^{\cdot}(t) \approx e_{\alpha\beta}^{(0)}(t) + \int k_\alpha k_\beta H(\mathbf{k}, \sigma_{11}, \sigma_{22}, \sigma_{12}; t) S(\mathbf{k}) d\mathbf{k}. \qquad (6)$$

Here $e_{\alpha\beta}^{(0)}(t)$ — strain of medium, calculated without taking into account initial inaccuracies; $S(k)$ - the spectral density of initial inaccuracies (function of the wave vector k); $H(t)$ - the elementary function of creep, which depends on nominal stresses/voltages $\sigma_{\alpha\beta}$ and wave vector k. The general expression for $H(t)$ is given to [17]. Under some simplifying assumptions this expression will take the form:

$$H(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{k_\alpha k_\beta \sigma_{\alpha\beta} e^{\rho t} \frac{d\rho}{\rho}}{k_\alpha k_\beta k_\gamma k_\delta \psi' h^2 A_{\alpha\beta\gamma\delta}(\rho) + \frac{k_\alpha^2 G^*(\rho)}{1 - \psi'} + k_\alpha k_\beta \sigma_{\alpha\beta}} . \qquad (7)$$
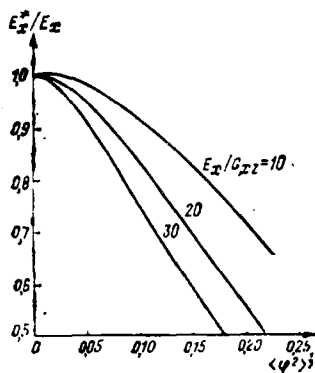
E_x^*/E_x

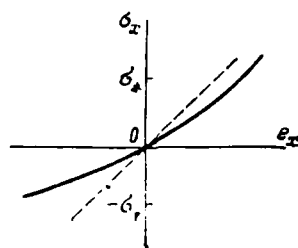Fig. 9.   Fig. 10.

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In formula (7) is used agreement about summation over "silent" indices and, furthermore, are introduced following designations: ψ' - coefficient of reinforcement; h - characteristic thickness of reinforcing layer; $\Lambda_{\alpha\beta\gamma\delta}$ (p) — viscoelastic operators. Corresponding to the flexural rigidities of the anisotropic reinforcing layers; G"(p) - viscoelastic translation operator for the bonding agent; p - parameter of the conversion of Laplace-Carson. Formulas (6) and (7) make it possible to obtain the series/row of quantitative and good-quality conclusions/derivations. One of them consists of the indication of the possibility of the continuous spectra of relaxation times in the reinforced materials, whose components possess discrete spectrum.

L. P. Khoroshun develops correlation theory of reinforced materials. In article [90] L. P. Khoroshun used to the task about the distribution of stresses and strains in stochastic medium the

method of moments/torques and he manufactured computations for the case of the strong correlation of the properties of components in one and two measurements. In this case pair correlations were considered.

V. V. Novozhilov [62] developed method of setting connection between macroscopic stresses and macroscopic strains for composite, which is found in macroscopically heterogeneous stressed state. In this case it turned out that into the relationships/ratios of connection the derivatives of macroscopic stresses and strains enter, i.e., the relationships/ratios indicated correspond to a certain model of the generalized theory of elasticity. To the development of statistical mechanics for structural-heterogeneous media, which considers plastic deformations and creep strains, are dedicated the works of Yu. I. Kadashevich and V. V. Novozhilov [37, 38].

3. Mechanics of the decomposition of composite materials.

Questions of decomposition of solid bodies belong to number of most urgent problems of contemporary mechanics. These questions are especially important for the artificial composite materials. Using the classical construction and engineering materials there are vast experimental data, which make it possible to carry out the engineering calculations, without turning to the theory of decomposition. Such data either are absent using the new materials or they are insufficient. Moreover, composite materials are usually projected/designed anew, in connection with which the theoretical

forecast of their strength it becomes completely necessary.

Study of strength of composite begins from study of strength of
its components. The majority of new composites is created on the
basis of high-strength filaments, filaments, filamentary crystals,
thin wires, etc., and also formed from these elements flat/plane and
three-dimensional/space structures. Vast experimental information
according to the mechanical properties of these elements is contained
in works [4, 11, 26].

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The dependence of rupture stress from the sizes/dimensions is one of
the essential features of high-strength thin elements (scale effect of
strength). For example, the strength of thin glass fibers noticeably
is reduced with an increase in length and diameter of filament. This
phenomenon satisfactorily is described within the framework of the
statistical theory of the decomposition of Weibull, which is based on
the concept of weak component/link [106]. According to one of the
simplest versions of this theory, the middle limit of the strength of
filament is defined as

$$\langle \sigma \rangle = \sigma_0 + \sigma_c \left(\frac{\Omega_0}{\Omega} \right)^{1/\alpha} \Gamma \left(1 + \frac{1}{\alpha} \right). \qquad (8)$$

Here σ_0 - minimum ultimate strength; σ_c — certain characteristic
stress/voltage; α - parameter of distribution of Weibull; Ω -
geometric characteristic of filament (for example, area of its cross
section, surface or volume); Ω_0 - standard value of this

characteristic. Statistical theory also explains the scatter of the limits of the strength of filaments. For the dispersion of ultimate strength the theory gives the formula

$$D\ [\sigma] = \sigma_0^2 \left(\frac{\Omega_0}{\Omega}\right)^{2/\alpha} \left[\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma^2\left(1+\frac{1}{\alpha}\right)\right]. \qquad (9)$$

Characteristics, entering formulas (8) and (9), are determined either on basis of model presentations about mechanism of decomposition, which uses, for example, model of brittle decomposition of Griffiths [96] or by statistical processing of results of tests. Fig. 11 depicts typical curves for the probability density $p(\sigma)$ of limit strengths σ with three different values Ω. The generalization of formulas (8) and (9) to the case, when strength they are defined simultaneously both by surface and three-dimensional/space defects, can be found in article [12].

Connecting/cementing materials (heat-activated polymers, resins, metals), as a rule, are isotropic. Their mechanical properties are determined from the common mechanical tests. However, for calculating the strength of composites it is necessary to know not only the internal strength (cohesion) of the connecting/cementing materials, but also adhesion strength (adhesion) of the connecting/cementing material with the filler. The mechanics of adhesive strength is developed still insufficiently. The effect of structural imperfections on the adhesion little is studied.

Forecast of strength of composite material with known mechanical

characteristics of components - difficult task. In the difference,
for example, from the bending characteristics, which characterize
connection/communication between the averaged stress fields and
deformations, strength characteristic depend substantially on the
effects at level h, i.e., at the level of grain, filament, etc.
Therefore the reliable calculated formulas for ultimate strength are
obtained only in the simplest cases of [42]. The elongation of
unidirectional fibrous material along the filaments will be such case.

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If the maximum deformation of bonding agent is greater than the
maximum deformation of filaments (Fig. 12), then tensile strength
along the filaments can be defined as

$$\sigma_{b_+} = \sigma_b{}' \left[\psi + (1 - \psi) \frac{E''}{E'} \right], \qquad (10)$$

where σ_b' — limit of the strength of the reinforcing material; ψ - its
relative volumetric content; E' and E" - intersecting the
moduli/modules of filler and connecting during the deformation, which
corresponds to the decomposition of filaments. The formulas of the
type (10) and their generalization extensively are used in the applied
calculations. If reinforcement is produced by short filaments, then
appears the need for the account of the heterogeneity of stress
distributions along the length of filament. In this case the concept
about the effective fiber length is utilized [11, 42, 101]. The
length of the sections of the filaments, which prove to be
insufficiently stressed, depends on the relationship/ratio between the
rigidity of filler and bonding agent. If this relation is

sufficiently great, and the cuts/sections of filaments sufficiently

short, then the strength of composite will be substantially less than

the value, determined from the formulas of the type (10). The bending

of filaments, their inaccurate orientation and other structural

imperfections also lead to reduction in the strength.

Compressive strain of composites along filaments can be

accompanied by loss of stability of reinforcing elements. In this

case in the bonding agent also on the contact surface between the

bonding agent and the filler appear the considerable

stresses/voltages, which lead to the disturbance/breakdown of adhesion

and in the final analysis to breaking of composite. On this base were

made the propositions estimate the strength of fibrous and laminar

composites during the compression from the considerations of the local

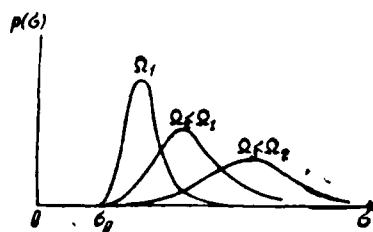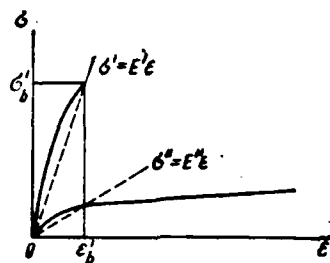stability of the reinforcing elements.

Fig. 11.   Fig. 12.

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Under some simplifying assumptions compression strength is estimated as

$$\sigma_{b-} \sim \frac{G''}{1-\psi},\qquad (11)$$

where by G" - the modulus of shift/shear of bonding material. The more careful calculation of the local stability of laminar composite during the compression was given by L. Pomazi [68, 69], who indicated the boundaries/interfaces of the application of the approximation formulas. To the analogous relationships/ratios we come, examining the local bulge of the laminar composite, whose reinforcing elements have low initial inaccuracies [17]. Some models of decomposition by interlamination breakaway and interlaminar shear are described in works [26, 42, 54].

Strength with simplest forms of macroscopic deformation was examined until now. The construction of the theory of the decomposition of composites in the complicated stressed state represents serious difficulty. The promising way was noted by Yu. N.

Rabotnov [73], who examined the conditions of the limiting condition of composites with the components, which possess upurgo-plastic properties. I. I. Gol'denblat and co-authors [5, 33] analyzed the phenomenological equations of boundary surfaces for the anisotropic materials and they gave the generalized criterion of decomposition, in which are utilized the mixed invariants of the stress tensor and some tensors, which characterize the properties of composites. In book [5] it is possible to find also the experimental data about the strength of glass-fiber-reinforced plastics under the conditions of the complicated stressed state. Some additional information is contained in the first chapter of this work.

Considerable statistical straggling of parameters of strength of many composites makes it necessary to turn to statistical theories of decomposition. By these theories must be considered the random distribution of structural imperfections, the statistical straggling of the strength of components, and also the random character of the accumulation of damages and development of cracks in the composites.

There are numerous attempts at application to composites of statistical theory of brittle decomposition, based on concept of weak component/link. As an example let us point out to the work of S. V. Sorensen and V. S. Strelyayev [54, 77], which utilized this theory for describing of scale factor and variability of the strength of unidirectional glass-fiber-reinforced plastic. Yu. N. Novichkov [59] used this theory for the forecast of the strength of the filled epoxy

compound.   In both cases is obtained satisfactory agreement of

calculation and experiment.   At the same time it is necessary to

recognize that the reinforced plastics of fibrous and laminar

structure are systems with the high degree of redundancy.

Observations of the process of the decomposition of samples from the

reinforced composites show that the first stage of decomposition

usually carries the retarded character.   Separate small cracks and

breakage of the separate reinforcing elements yet do not lead to the

formation of main-line crack.   The latter appears only after the

damages, scattered in the volume of composite, reach a certain

critical density.   For describing this phenomenon usually is utilized

the model of Daniels [95] (for example, see works [11, 26, 42, 54]).

In connection with this let us examine the model indicated in more

detail.

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Let the binder of n parallel filaments of identical section/cut
be dilated/extended by quasi-static force N. We will consider that
the filaments between themselves do not interact and that the load is
equally distributed between the undestroyed filaments. Let us
designate stresses in these filaments by σ. Rupture stress for the
taken at random filament - random variable; the function of the
distribution of this value let us designate through $F(\sigma)$. According
to Daniels, the force which affects on the truss in the process of the
decomposition of filaments is calculated as follows:

$$N = n\sigma\,[1-F(\sigma)]. \tag{12}$$

The collapsing force and number of filaments, whose breakage
leads to decomposition of truss as a whole, are determined from
condition that

$$N(\sigma) = \text{макс}. \tag{13}$$

Model of Daniels possesses essential shortcomings, which can be
illustrated based on following example. Let the destructive fiber
stress be evenly distributed on interval $\sigma_l \leqslant \sigma \leqslant \sigma_u$. then we obtain
according to formula (12):

$$N = \begin{cases} n\sigma & (\sigma \leqslant \sigma_l); \\ n\sigma\left(1 - \dfrac{\sigma-\sigma_l}{\sigma_u-\sigma_l}\right) & (\sigma_l < \sigma \leqslant \sigma_u). \end{cases} \tag{14}$$

If $\sigma_u \geqslant 2\sigma_l$, then it is obtained taking into account (13) and (14),

that rupture stress for truss of filaments is equal to smallest rupture stress in general population of filaments, i.e. $N=n\sigma_l$ (Fig. 13). With any final n this value proves to be less than the value determined according to the theory of weak component/link. Thus, in the example examined the model of Daniels not only does not consider the reserving of strength in the composite, but also gives the clearly decreased value of rupture stress. However, in the literature [54, 77] it is possible to find the series/row of the examples, when the model of Daniels satisfactorily agrees with the experimental data.

The decomposition of the composite is a random process, which develops in time. Therefore, the correct description of the phenomenon of decomposition is possible only on the basis of the theory of random processes. At level h composite can be considered as the system, which consists of the large number of discrete elements.
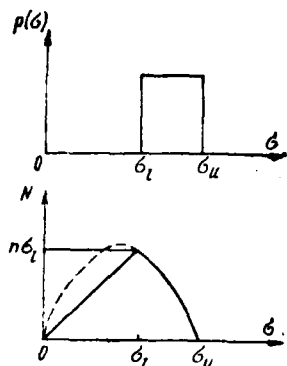
Fig. 13.

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The accumulation of damages and the development of main-line cracks occurs due to the decomposition of the weakest elements, and also the elements which entered the region of concentration at the front of crack. As external this manifestation serves granulated and fibrous appearance of fracture upon the short-term, prolonged and fatigue failure, the intermittent advance of crack, etc.

Method stochastic description of decomposition of system, which consists of n elements, consists of following. Let us introduce the n-dimensional random process:

$$Z(t) = \{Z_1(t), Z_2(t), \dots, Z_n(t)\}, \tag{15}$$

where $Z_j(t)$ — the random functions of time t. The range of change in these functions let us select as follows:

$$0 \leqslant z_j(t) \leqslant 1. \tag{16}$$

In this case value $z_j = 0$ corresponds to sound element, and value $z_j = 1$ — to completely destroyed element. Joint probability density p(z, t) for the components of vector process (15) satisfies the kinetic equation

$$\frac{\partial p(z;t)}{\partial t} = H\left[p(z;t), s(t), T(t)\right]. \qquad (17)$$

where H – certain operator; s – parameter (or the set of the parameters) of external load; T – parameter, which characterizes temperature field. Equation (17) is solved under the initial conditions, which correspond to the state of system with t=0.

Equation (17) corresponds to system with finite number of elements and continuum of states (16). It proves to be differential or integrodifferential equation relative to function n+1 of the independent variables. The numerical realization of this model even with very small numbers n presents serious difficulties. We will obtain a considerable simplification, after assuming that the elements can be found only in the finite number of states. For example, distinguishing only two states of element – whole and destroyed, let us assume that the components of process (15) take two values. Value $z_j = 0$, corresponds to whole element, value $z_j = 1$ — to destroyed element. Probability to reveal/detect the j element in the whole state let us designate through $P_j(t)$.

Let us introduce n-dimensional vector process of P(t) with components $P_1(t)$, $P_2(t)$, ..., $P_n(t)$. Instead of equation (17) we will obtain the system of the ordinary differential equations:

$$\frac{dP}{dt} = H \left[\mathbf{p}\left(t\right), \mathbf{s}\left(t\right), T\left(t\right) \right]. \qquad (18)$$

The task lies in the fact that to construct operator H on basis of model presentations about interaction between elements of composite and statistical data about distribution of strength of elements. Simplest description we will obtain assuming that the decomposition is Markov process.

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Then, for example, in equation (18) operator H comes to the multiplication of vector P(t) by a certain matrix.

Generally, the theory of random Markov processes gives large number of convenient mathematical models. Are such, for example, the processes of pure/clean death, birth and death processes, processes of the random walk, etc. These mathematical models extensively are used in theoretical physics, chemical kinetics, theory of reliability, theoretical biology, mathematical saving and in other regions of applied mathematics. Many of these processes can be interpreted and as the mathematical models of decomposition, and the large number of free constants and arbitrary functions makes it possible to satisfy virtually any sufficiently limited totality of experimental data. The first attempt to describe decomposition as one-dimensional Markov process belongs, apparently, to Yokobori [107], and as the multidimensional process of pure/clean death - to Coleman [92].

References to some subsequent works can be found in article [40].

Assumption about Markov behavior of process is not too rigid.
Markov process is the random process, for which the probabilistic
relationships/ratios, which determine the future, depend on the state
reached at the given moment, i.e., they do not depend on prehistory.
It would seem that in this case the effects of plasticity,
aftereffect, accumulation of damages, etc. completely are eliminated.
In actuality freedom in the selection of the fact that we call the
state of system, makes it possible to sufficiently fully describe
these effects. But this each time will require the corresponding
increase in the dimensionality of phase space.

Unfortunately, if we turn to real bodies from composites, then
the dimensionality of phase space proves to be is very great.
Therefore it is possible to speak only about the mathematical model of
the process of decomposition, which transmits the basic good-quality
features of process.

We spoke about the decomposition generally. Until now, without
isolating development stage of main-line crack. For stochastic
description of main-line crack - the connected set of those destroyed
elements - there will be required a certain further elaboration. Let
us show this based on the example to the one-dimensional model, which
consists of the n elements. As its physical analog can serve a flat
single-layer sample from the unidirectional fibrous composite,

containing n of filaments. Let us calculate the number of possible states of this model, by assuming that each element can be found only in one of two states (whole or destroyed). This corresponds to the assumption that not one element can be destroyed in more than one place and that prelimir_ry damages are not taken into consideration. Then the number of states of system m will be equal to the number of all possible combinations of n elements, i.e., $m = 2^n$ (Fig. 14a). Even for n=10 number of states proves to be so great that the model cannot be realized in the contemporary electronic computers.

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Let the main-line crack be the only one, but it can be conceived by the decomposition of any of the n elements (Fig. 14b). Then $m \approx n^3$. But if the side or internal concentrators serve as sources of the main-line cracks or (on the condition) main-line cracks can be conceived only on the surface (Fig. 14c), then $m \approx n^2$.

Further reduction of phase space is possible, if we are restricted to examination of sole crack, which is incipient from surface or in concentrator (Fig. 14d). In this case of m=n+1. For this case let us give equations relative to probabilities $P_\mu(t) = P(E_\mu; t)$. Here E_μ — state, which corresponds to the advance of crack on μ elements. Equation takes the form:

$$\frac{dP_\mu(t)}{dt} = \sum_{v=0}^{n} a_{\mu v}(t) S_v(t) \qquad (19)$$
$$(\mu = 0, 1, 2, \ldots, n),$$

where $a_{\mu v}$ — intensity of transition/transfer. The latter are

expressed by means of the maximum operations through the probabilities of transition of system from one state into another:

$$a_{\mu\nu}(t) = \lim_{\Delta t \to 0} \frac{P(E_\nu \cdot E_\mu; t \to t+\Delta t)}{\Delta t}. \qquad (20)$$

Diagonal matrix elements $a_{\mu\nu}$ are defined as

$$a_{\mu\mu}(t) = 1 - \sum_{\rho=0}^{n} a_{\rho\mu}(t) \quad (\rho \neq \mu). \qquad (21)$$

If the probability of "healing" of main-line crack is excluded by hypothesis, then with $\mu < \nu$ everything $a_{\mu\nu} = 0$.

System of equations (19) is integrated under initial conditions, which characterize state of system with t=0. For example, if the initial state of system sound, then $P_0(0)=1$, and everything else $P_\mu(0) = 0$. If in the initial state is a concentrator - crack, which contains ν_0 elements, then $P_{\nu_0}(0) = 1$, and everything else $P_\mu(0) = 0$.
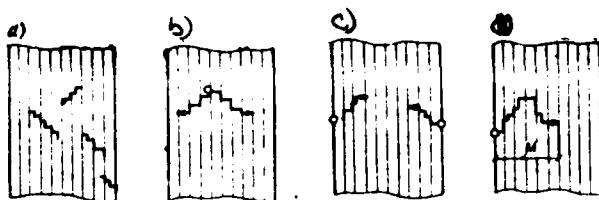
Fig. 14.

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Above there has already been discussed the fact that it is possible to attain by increase in dimensionality of phase space satisfactoriness of Markov model for broad class of physical phenomena. Contemporary electronic computers make it possible to solve equations for the discrete/digital Markov systems with the number of states, which is of the order of 100. Question arises, along which way to more advantageous go in order to achieve the more complete description of system. It seems to us that it is more expedient to use models with stochastic heredity, than to increase the dimensionality of phase space. Instead of equation (19) we come to the system of the integrodifferential equations:

$$\frac{dP_\mu(t)}{dt} = \sum_{v=0}^{n} a_{\mu v}(s, T) P_v(t) + \sum_{v=0}^{n} \int_0^t h_{\mu v}(t - \tau; s, T) P_v(\tau) d\tau \qquad (22)$$

$$(\mu = 0, 1, 2, \ldots, n).$$

In equations (22) s=s(t), T=T(t), and $h_{\mu v}(t-\tau)$ — nucleus of type of Volterra, who considers the effect of the prehistory of loading. But processes with stochastic consequence of the type of the processes of death are not yet studied sufficiently. Completely there are no

methods of statistical processing in connection with similar

processes. Meanwhile for the development the theories of the

prolonged and fatigue failure of model with stochastic heredity of the

type (22) can prove to be very promising.

For computing of intensities $a_{\mu\nu}(t)$ and nuclei $h_{\mu\nu}(t-\tau)$ is

necessary further concrete definition of model. ·The model,

that considering two mechanisms of destruction - short-term
destruction and delayed fracture, was examined in article [24].
The series/row of examples was designed in the electronic computer.
The greatest number of elements was $n=50$, which in the case of one-way
crack propagation corresponded to system of equations (19), that
contains 51

unknown function. The load case, which corresponds to sample testing

in the rigid testing machine with the constant velocity of strain, was

examined. The example to the dependences of probabilities $P_v(s)$ on the

parameter of strain s is represented in Fig. 15. Furthermore, Fig. 16

gives the curves, which show a change in the mathematical expectation

<N> of the total force N with different ν_0. Here it is not possible

to enter into a detailed discussion of all the results. We will be

restricted to indication that the introduction of concentrators

increases the probability of observing the small quantity of destroyed

elements with the increasing load. In this case the decomposition

acquires more viscous nature. Is of interest dependence <N> on

parameter s. For the case $\nu_0=0$ maximum value <N> composes 33.7. At

the same time application of a model of weak component/link gives

$<N> = 25,5$ This disagreement is completely natural, if one considers

that in Weibull's model the decomposition of system begins upon the

decomposition of the weakest element in the random sampling from the n

elements.

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But if we to the example in question use the model of Daniels, then the maximum value <N> proves to be equal to 25, i.e. it corresponds to the bearing capacity of the weakest element in the general population.

Introduced model describes effect of redundancy in composites (for example, in laminar and fibrous materials), on in more specific and more flexible form than this is done in theory of reliability. The essential feature of this model is the fact that the consecutive decomposition of the elements, which stand on the way of crack, is treated as random process with the discrete set of states. The known effects of the strain/work hardening brittle materials due to lamination [93, 94] are described with the aid of the model proposed.

4. Calculation of constructions/designs from the composite materials.

Basic difference in structural mechanics of constructions/designs from composite materials from its classical sections consists of need for considering essential anisotropy of mechanical characteristics of the material. Furthermore, frequently there is required an account of structural heterogeneity, those the examination of the stress fields and strains on the level of grain, filament, etc. Finally, if components possess the sufficiently expressed flow properties, then these properties will possess a composite. Consequently, appears the

need for the account of temporary/time effects.

The theory of elasticity of anisotropic bodies and structural mechanics of constructions/designs from anisotropic materials based on it are developed in sufficient detail [2, 3, 45, 46]. The presentation of some questions of theory in application to composite materials can be found in works [5, 8, 63, 72, 83, 86].

Majority of anisotropic composite materials of laminar and fibrous structure is intended for transmission of flux of forces in some preferred directions.
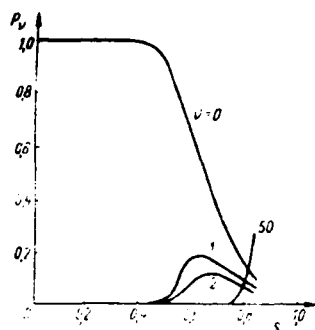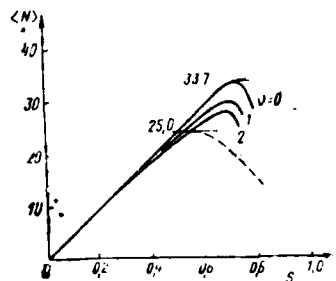
Fig. 15   Fig. 16.

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Material is distributed in such a way as to ensure the greatest

strength and rigidity in these directions. But this is reached due to

reduction in the strength and rigidity in the direction, orthogonal of

the axis/axle (plane, surface) of reinforcement. As an example let us

point out to the low resistance of laminar glass-fiber-reinforced

plastics to elongation across layers and to interlaminar shear. The

calculation of constructions/designs must ensure a sufficient

reliability with respect to these forms of strain. This requires the

increased attention taking into consideration of anisotropy.

One of the simplest tasks, where the effect of anisotropy proves

to be essential, is the task about lateral flexure of rod from

unidirectional composite is forces applied along the normal to

direction of reinforcement (Fig. 17). In the rods with the compact

cross section shearing stresses, which affect along the reinforcement,

are of the order H/Λ. Here H is the characteristic height of the

section; Λ - the reference length of the rod. In view of the low
strength of composite with interlaminar shear these stresses/voltages
must be considered taking into account the strength even with the the
low H/Λ. Anisotropy must be considered also taking into account the
rigidity. In the rods from the isotropic material the effect of
shifts/shears on the sagging/deflection of rod is of the order H^2/Λ^2
in comparison with one. The correction for shift/shear is so low for
the stems which in the engineering is not considered. In the case of
rod from the unidirectional composite the correction for shift/shear
is of the order of the parameter η, where

$$\eta = \frac{H^2}{\Lambda^2} \cdot \frac{E_x}{G_{xz}} . \qquad (23)$$

Here E_x — modulus of composite in the direction of reinforcement; G_{xz} —
modulus of interlaminar shear. Since $E_x \gg G_{xz}$, the parameter η can be
of the order of one and more.

        Fig. 18 gives dependence [86] of ratio of sagging/deflection w,
calculated taking into account interlaminar shears, to
sagging/deflection w_0, calculated according to common formulas of
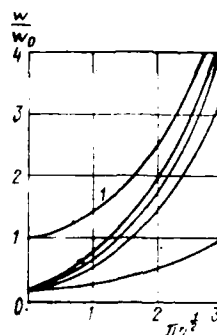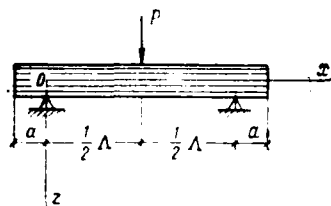strength of materials.

Fig. 17.   Fig. 18.

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Along the axis of abscissas is plotted the value proportional to

square root from parameter (23). Curve 1 corresponds to the rod

supported on the ends. Remaining curves are constructed for the

diverse variants of jamming bearing sections/cuts. As can be seen

from Fig. 14, the effect of shifts/shears is so great which to

disregard it is possible only for the very stems. For the

glass-fiber-reinforced plastics the relation of moduli/modules $E_x G_{xz}$

is of the order of 10, while for the boron fiber-reinforced plastics

and a carbon-fiber reinforced plastic-order 100.

Attention is drawn to the strong dependence of strains on the

method of realization of boundary conditions. This phenomenon,

typical for the constructions/designs from the strongly anisotropic

materials, is the specific manifestation of St. Venant principle.

According to the principle of hay-venans for the isotropic elastic

medium, the disturbances/perturbations, introduced by the

self-balanced system of forces, rapidly attenuate at the distances

from the perturbation source, greater in comparison with the

significant dimension of source H (Fig. 19a). In the case of the

anisotropic medium of disturbance/perturbation they attenuate

dissimilarly in different directions. In the direction of the

greatest rigidity of disturbance/perturbation they attenuate more

slowly, while in the direction it is the smallest rigidity - more

rapid. As a result the region of noticeable

disturbances/perturbations is extracted in the direction of the

greatest rigidity (Fig. 19b). The significant dimension of the region

of disturbances/perturbations in this direction is of the order

$$\Lambda \sim H \left(\frac{E_x}{G_{xz}} \right)^{1/2} . \tag{24}$$

This phenomenon is understandable from examination of structure

of composite: it is hindered/hampered in view of low rigidity of

bonding agent to shift/shear of redistribution of forces between

reinforcing elements.

Calculation of constructions/designs from composites is produced

either on basis of theory of elasticity, viscoelasticity and the like

for anisotropic media or on basis of theory of reinforced media,

developed specially in application to composite materials. The

calculation of rods, plates and rings from the composites is in detail

presented in the book of Yu. M. Tarnopol'skiy and A. V. Roses [86],

that contains vast reference material and detailed bibliography.

Therefore here we will be restricted to the indication of only some

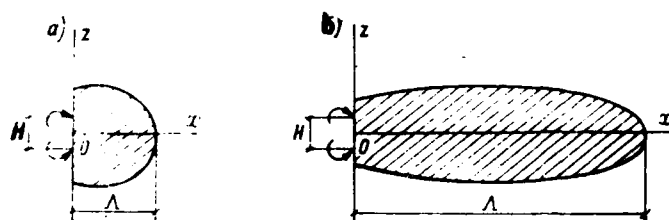works on the theory of shells of composite materials.

Fig. 19.

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Version of this theory was proposed by V. V. Bolotin and V. N.

Moskalenko in the report at the All-Union conference on the theory of

shells and plates (Moscow, January 1965). The transactions of this

conference, as is known, were not published. It is presented the

content of this report, following article [20]. Let us assume as the

basis of the equation of the theory of reinforced media [15]. Let us

assume that are satisfied all conditions, which allow the application

of the principle of smoothing, i.e., a transfer to the equivalent

quasi-homogeneous medium. Following the basic idea of the theory of

shells, let us select certain median surface and, utilizing totality

of kinematic and static hypotheses, let us reduce three-dimensional

task to two- dimensional for some functions on median surface. Let us

assume that the deformation of shell with a sufficient

precision/accuracy is described with the aid of six

function-displacements of the points of the middle surface v_α and w,

angles of rotation of standards/normals θ_α and function of the stretch

deformation of standards/normals ξ. Here index α take values of 1, 2.

The application of variation principle reduces to the system of

equations of equilibrium relative to the tensor of efforts/forces in

median surface, the tensor of the moments/torques of general/common

bending, tensor of the bending moments of the reinforcing layers and

tensor of bimoments, connected with the stretch deformation of

standards/normals. The enumerated tensors are connected with the

appropriate strain tensors with some relationships/ratios of

elasticity. The equations of equilibrium, expressed through functions

v_a, w, θ_x and ξ, are reduced to the resolving equation of the sixteenth

order. At each point of the contour/outline of median surface are

placed eight boundary conditions. Failure of the account of moment

effects in the reinforcing layers depresses the order of the resolving

equation by two. If, furthermore, to forego the account of the

stretch deformations of standards/normals, then the order is reduced

to six. After these simplifications the system of equations

corresponds to a certain anisotropic version of the refined theory of

shells according to Timoshenko. In article [20] is given the

series/row of examples of the application of the theory presented.

The contribution of moment effects, deformation of standards/normals

and transverse shifts/shears to amount of deflection of plate from the

laminar composite is evaluated. The character of edge effects in the

circular cylindrical shell is studied. Besides the edge effect of

Love, known from the classical theory of shells, and the edge effect

of Reissner, caused by transverse shifts/shears, the appearance of

edge effects of other types is possible. It is such, for example, the

relaxation effect of moments/torques in the reinforcing layers, the

relaxation effect of the stretch deformations of standards/normals,

and also the mixed effects.

Very general/common version of equations of theory of multilayer shells is developed by V. N. Moskalenko and Yu. N. Novichkov [52]. Examined were shells which consist of the alternating layers of that reinforcing and the bonding agent of materials. For the reinforcing layers was accepted Kirchhoff-Love hypothesis, for the bonding agents a layer-assumption about the smallness of all deformations, except transverse shears and elongation of the normals.

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Essential difference in the theory proposed by V. N. Moskalenko and Yu. N. Novichkov from other variations of the theory of laminar shells lies in the fact that in this theory the dependence of metric properties on the number of a layer is considered. In other words, the vector and tensor fields, which describe the stress-strain state in the shell, relate not to the middle shell as a whole, but to those its corresponding to layers. This is achieved by the introduction of the translation operators of tensor objects from one surface to another. The derived equations are applicable to the shells, whose thickness it is compared with their characteristic radius. Using to the equations derived by V. N. Moskalenko and Yu. N. Novichkov and the principle of smoothing, we will obtain equations for the moment anisotropic elastic medium referred to the curvilinear coordinate system.

Transmission of forces in laminar composites was studied by V. V. Partsevskiy, V. N. Moskalenko [21, 53, 64], and others. In the article [21] the loading of half-space from the laminar composite during the arrangement of layers of orthogonally free surface is examined. As basis is assumed discrete model from work [16]. The effect of the parameters of composite on stress distribution in the bonding layers is studied. In works [64, 65] the results were used for solving the problem about the pressure of rigid die/stamp on the half-space from the laminar composite and about stress concentration in the vicinity of a deep cutout, arranged/located orthogonally to layers. The transmission of efforts/forces in the composite with the loading along the normal to layers is studied in article [52].

Large number of works is devoted to calculation for stability of rods, plates and shells of anisotropic materials [75, 76, 78, 79, 86]. Let us pause at some of them, in which are considered temporary/time effects. Ye. N. Sinitsyn [79] examined the task about the behavior of the flattened rods from the viscoelastic material on the assumption that the reinforcing material is ideally elastic, and bonding agent is standard linear viscoelastic medium. Task is examined taking into account geometric nonlinearity. Is examined the behavior of rods during the compression by the prescribed/assigned load with the loading via the approach of supports, and also during the heating. Work [78] examines also some tasks about the stability of plates, while in work [18] - tasks about the surface bulge of half-space from the linear viscoelastic half-space.

Dynamic tasks for constructions/designs from composites are studied still insufficiently. There is great interest in the development of the theory, which describes the dissipation of energy during the oscillations/vibrations of these constructions/designs. This theory is necessary for the forecast of the reaction of constructions/designs to the action of vibration loads, for the forecast of the sound-insulating properties, etc. In this region only just the first results [19] are obtained. Another region of dynamic theory - investigation of wave processes in the composite media. As is known [43], vibration, sonic and ultrasonic techniques extensively are used for diagnostics of composite materials and for the nondestructive tests.

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The application of these methods requires the information about the dependence between the properties of composite and the velocities of propagation of longitudinal, shear and Rayleigh waves. Some tasks about the propagation of will in the laminar media are examined by Yu. N. Novichkov [60, 61].

Constructions from artificial composites are very adequate/approaching object for applying principles of optimum planning. Specifically, here the idea of optimization can be realized to the end. Because of the large freedom during the selection of the mechanical characteristics of material, its anisotropy,

structural/design forms and sizes/dimensions of carriers. However, in this region it is made still very little. Developed most in detail are methods of the optimization of shells and tubes/pipes from the glass-fiber-reinforced plastics, which work under the pressure. These constructions/designs are usually formed by the method of continuous winding from the glass filaments or the fiberglass tape. The question about the determination of the optimum parameters of bonding agent and filler, optimum arrangement of layers and optimum initial tension of glass filaments or fiberglass tape (latter connected with the selection of the system of initial stresses/voltages in the article) arises. In practice optimization conditions are usually replaced by some less rigorous conditions. For example, it is required that the carrying layers would be uniformly strong with respect to normal stresses, but shearing stresses in the bonding agent did not exceed limiting value so that the trajectories of principle normal stresses would coincide with the trajectories of glass filaments, etc. Some information about the works from the selection of the rational systems of the reinforcement of shells, which are located under the pressure, can be found in [100, 102, 104].

New tasks of structural mechanics appear in connection with technology of structural design from composites. For the certainty we will speak about the composites on the basis of polymeric resins. It is known that the chemical shrinkage of epoxy resins during the polymerization is approximately 2.5%, and in polyester resins it reaches to 15%. The polymerization of bonding agent usually is

produced at elevated temperatures. In the process of hardening the bonding agent in the composite appears the system of internal stresses/voltages, which partly has temperature origin, partly caused by the chemical shrinkage of the bonding agent. In the glass-fiber-reinforced plastics, where the coefficients of the thermal expansion of components can differ 10 times and more, initial stresses/voltages can have a considerable effect on general/common strength. Sometimes article is cracked from the disturbance/breakdown of adhesion already in the process cooling after the polymerization of bonding agent.

Two types of initial stresses/voltages are distinguished. The first type - these are the local stresses, whose scale is of the order of the transverse size/dimension of filament, the thickness of the layer, etc. For the definition of these stresses/voltages composite should be considered as substantially heterogeneous material. To the study of local shrinkage and thermal stresses are dedicated the works of A. L. Abibov and G. A. Molodtsov [1], and G. A. van Fo Fy [29, 89], etc.

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The second type of initial stresses/voltages - these are macroscopic stresses/voltages. As an example they can serve stresses/voltages in the tubes/pipes and shells of glass-fiber-reinforced plastic, which are formed by the method of winding. The process of the emergence of initial stresses/voltages is studied in article [22]. In this case it

is shown that with the small tensions of glass filaments, fiberglass tape and the like the initial stresses/voltages in essence appear for the stage of colling of the prepared article from the temperature of polymerization to the operational temperature. This conclusion is confirmed by numerous experiments [9, 10]. For the illustration given in Figs. 20 and 21 are the results of determining the residual stresses from the method of Zaks on the annuluses from the glass-fiber-reinforced plastic. Here $\xi=(r-r_1)/(r_2-r_1)$, r_1 - internal, r_2 - external radii of the ring. Fig. 20 gives the distribution of radial stresses (these stresses - stretching). Solid line is obtained by calculation according to the formulas from article [22]. Fig. 21 depicts the diagrams of circular stresses. Experimental curves are designated just as in Fig. 20; calculated curve is plotted/applied by solid line. The dependence of maximum radial stresses/voltages from ratio r_2/r_1 is shown on Fig. 22, and extreme circumferential stresses - on Fig. 23.
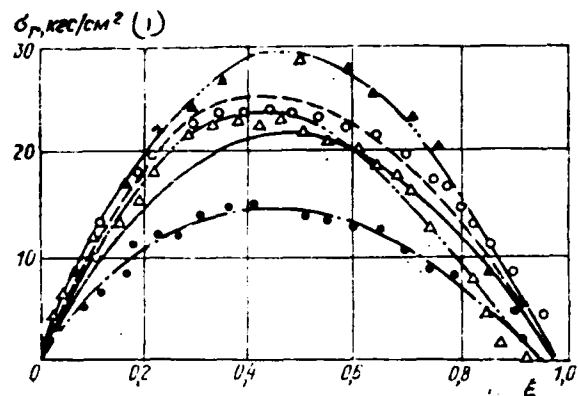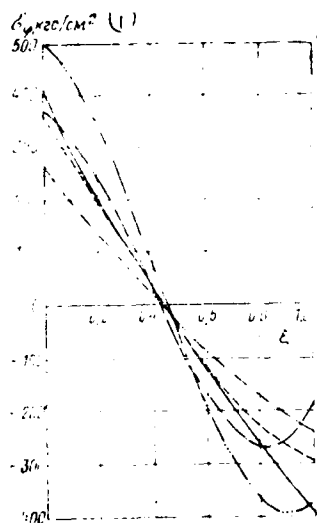
$\sigma_r, \kappa zc/cm^2$ (1)

Fig. 20.

Key: (1). kg/cm².

$\sigma_\varphi, \kappa zc/cm^2$ (1)

Fig. 21.

Key: (1). kg/cm².

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With large tensions in process of winding it is necessary to
consider their contribution to initial stresses/voltages. Calculation

in this case very becomes complicated, since initial tensions relax as
a result of the reduction of semi-finished product, filtration and
displacement of softened bonding agent, etc. Some of of these their
effects are taken into consideration in the articles of Yu. M.
Tarnopol'skiy and G. G. Portnov [82], V. L. Blagonadezhin with
co-authors [9].

Large quantity of new tasks appears in connection with
substantiation and development of methods for testing materials and
constructions/designs from composites. Above has already been
discussed the special features of St. Venant principle in application
to strongly anisotropic materials. These special features make it
necessary to focus increased attention on the method of the
Methods for sample testing from the laminar and fibrous
transmission of efforts/forces on samples

glass-fiber-reinforced plastics to the elongation and the bending were
examined by Yu. M. Tarnopol'skiy with co-authors [80, 81], the method
for testing for torsion - by V. P. Nikolayev and Yu. N. Novichkov
[56]. The series/row of works is dedicated to the tests of annular
samples on elongation and compression. Basic task here consists of
the estimation of error, which appear with the different methods of
the imitation of hydrostatic pressure. The survey/coverage of these
methods can be found in the book V. B. Meshcheryakova, A. K.
Sborovskiy and A. Ya. Goldman [51]. The method of elongation and
compression of annular samples with the aid of the semidisks, the
sectors and the like was studied by V. V. Partsevskiy [66, 67]. In
particular, there was taken into account the effect of the initial

clearance between the semidisks on stress distribution in the sample, and a question about the number of sectors, necessary for obtaining sufficiently uniform stress field, is also examined.

It would be possible to considerably expand the list of scientific directions and the results reached. The works which are conducted at the Moscow State University are illuminated in the book of P. M. Ogibalov and Yu. V. Suvorova [63], at the Institute of Mechanics of Polymers - in books o A. K. Malmeyster et al. [43, 50, 83, 86, 87], in the Institute of mechanics and the Institute of the problems of materials science of the Academy of Sciences of UkrSSR [Ukrainian SSR] - in book [89], at the Institute of Chemical Physicists - in King a. L. Rabinovich [72], in the institute of engineering science - in the book of S. V. Sorensen, etc. [54].
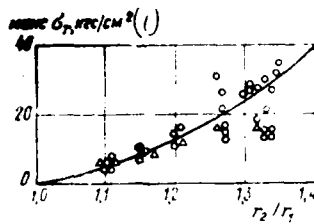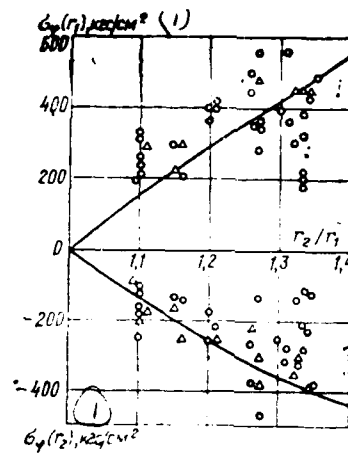
Fig. 22   Fig. 23.

Fig. 22. Key: (1).   kg/cm².

Fig. 23. Key: (1).   kg/cm².

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At present to the number of the most urgent/most actual directions, in our opinion, the mechanics of the decomposition of composites and constructions/designs of them and the theory of optimum planning belongs. Already during this decade it is possible to expect the wide acceptance of new materials in the building. In connection with this the problem of forecasting reliability and service life from the new composite materials becomes urgent/actual. It is necessary to study already this problem in the nearest time.

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